Exercise 54

Find $f^{(n)}(x)$ if f(x) = 1/(2-x).

Solution

Rewrite the function in a more convenient form.

$$f(x) = (2 - x)^{-1}$$

Take the derivative of f(x) several times and try to discover a pattern.

$$f'(x) = (-1)(2-x)^{-2} \cdot \frac{d}{dx}(2-x) = (-1)(2-x)^{-2} \cdot (-1) = (2-x)^{-2}$$

$$f''(x) = (-2)(2-x)^{-3} \cdot \frac{d}{dx}(2-x) = (-2)(2-x)^{-3} \cdot (-1) = 2(2-x)^{-3}$$

$$f'''(x) = 2(-3)(2-x)^{-4} \cdot \frac{d}{dx}(2-x) = 2(-3)(2-x)^{-4} \cdot (-1) = 2(3)(2-x)^{-4}$$

$$f^{(4)}(x) = 2(3)(-4)(2-x)^{-5} \cdot \frac{d}{dx}(2-x) = 2(3)(-4)(2-x)^{-5} \cdot (-1) = 2(3)(4)(2-x)^{-5}$$

$$\vdots$$

$$f^{(n)}(x) = n!(2-x)^{-(n+1)} = \frac{n!}{(2-x)^{n+1}}$$